Particle Physics - Exercises (Series 2)

① Prove that the α_i, β are hermitian, traceless of even dimensionality, with eigenvalues ± 1.

② Show that $\{\alpha_i, \alpha_j\} = \{\alpha_i, \beta\} = 0$, $\alpha_i^2 = \beta^2 = 1$ relations are preserved after

$$\alpha_i' = U \alpha_i U^{-1}$$
$$\beta' = U \beta U^{-1}$$
, U unitary matrix

③ Show $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu}$

and $\gamma^{\mu\dagger} = \gamma^0 \gamma^\mu \gamma^0$

④ Operate with $\gamma^\nu \partial_\nu$ on Dirac's equation and show that each of the four components satisfies the Klein-Gordon equation $(\Box + m^2)\psi_i = 0$.

⑤ Show that the following relation is true:

$$u^{(r)\dagger} u^{(s)} = 0 \quad r \neq s \quad \text{(orthogonality)}$$

⑥ Show that η (helicity) and H commute.

⑦ Take the conjugate equation of Dirac and try to relate ψ with the Dirac equation for a positron.

⑧ Starting from $(\not{p} - m)u = 0$ derive the equation for spinor $\bar{u}$, $\bar{u}(\not{p} - m) = 0$. Do the same for spinor $\bar{v}$ starting from $(\not{p} + m)v = 0$.

Series 2 (cont)

9) Show that:

$$\sum_{s=1,2} u^{(s)}_{(p)} \bar{u}^{(s)}_{(p)} = \not{p} - m$$

10) Solve Dirac for the case $m=0$.

11) Show that $\bar{\psi}\gamma^{\mu}\psi$ transforms as a four-vector (Lorentz transformation).