Particle Physics

Exercises (1st series)

1. Show that the non-homogeneous Maxwell equations can be written as:

$$\partial_\mu \mathrm{F}^{\mu\nu} = \mathrm{J}^\nu.$$

   Use this formula to show charge conservation.

2. Show that the homogeneous Maxwell equations can be written as:

$$\partial^\mu \mathrm{F}^{\nu\rho} + \partial^\rho \mathrm{F}^{\mu\nu} + \partial^\nu \mathrm{F}^{\rho\mu} = 0$$

3. Show that the derivative $\frac{\partial}{\partial \mathrm{X}^\mu} = (\partial_\mathrm{t}, \nabla)$ transforms as $\mathrm{X}_\mu = (\mathrm{ct}, -\overrightarrow{x})$ and that the derivative $\frac{\partial}{\partial \mathrm{X}_\mu} = (\partial_\mathrm{t}, -\nabla)$ transforms as $\mathrm{X}^\mu = (\mathrm{ct}, \overrightarrow{x})$.

4. Consider a Lorentz transformation in which the new frame (primed coordinates) moves with velocity υ along the z axis of the original frame (unprimed coordinates). For such a Lorentz "boost", show that:

$$\mathrm{ct}' = \cosh\theta \cdot \mathrm{ct} - \sinh\theta \cdot \mathrm{z},$$
$$\mathrm{z}' = -\sinh\theta \cdot \mathrm{ct} + \cosh\theta \cdot \mathrm{z},$$

   with x and y unchanged; here, $\tanh\theta = \frac{\upsilon}{\mathrm{c}}$. As $\cos \mathrm{i}\theta = \cosh\theta$ and $\sin \mathrm{i}\theta = \mathrm{i}\sinh\theta$ we see that the Lorentz transformation may be regarded as a rotation through an imaginary angle $\mathrm{i}\theta$ in the $\mathrm{ict} - \mathrm{z}$ plane.

5. Show that Schrodinger equation remains the same under a simultaneous gauge tranformation of the electromagnetic potentials and a local phase transformation of the wavefuntion.